Laser tracking interferometer system based on trilateration and a restriction for the position of its laser trackers

Toshiyuki Takatsuji\textsuperscript{a}, Yoshihiko Koseki\textsuperscript{b}, Mitsuo Goto\textsuperscript{a}, Tomizo Kurosawa\textsuperscript{a}, and Yoshihisa Tanimura\textsuperscript{a}

\textsuperscript{a}National Research Laboratory of Metrology, 1-1-4 Umezono, Tsukuba, Ibaraki 305-8563, Japan
\textsuperscript{b}Mechanical Engineering Laboratory, 1-2 Namiki, Tsukuba, Ibaraki 305-8564, Japan

ABSTRACT

To measure three dimensional coordinate we have been developing a laser tracking interferometer system (LTS). Four laser interferometers chase the movement of a target cat's eye and measure the change in distance between them. The position of the cat's eye is determined from the measured distances based on the principle of trilateration. Taking advantage of measurement redundancy produced by the fourth tracker, the position of the trackers and the initial position of the cat's eye can be estimated by a self-calibration algorithm. A restriction on the arrangement of the laser trackers to perform the self-calibration algorithm is theoretically studied. Finally a preliminary experiment was made to show the measurement error of about 40\,\mu m for a 1 m measurement.

Keywords: coordinate measurement, laser tracking, trilateration, laser interferometer, cat's eye retroreflector

1. INTRODUCTION

Coordinate measuring machines (CMMs) is a versatile measuring instrument and has been widely used in various fields.\textsuperscript{1} CMMs can perform very accurate measurement, nevertheless measurement errors are naturally included in its measurement results. One of the reason of this fact is that the mechanical structure of a CMM does not satisfy Abbé's principle.

In National Research Laboratory of Metrology (NRLM) of Japan, we have been developing a laser tracking interferometer system (LTS).\textsuperscript{2} In LTS measurement, a cat's eye retroreflector is attached instead of a probe of a CMM. Multiple laser interferometers each of which is mounted on a two-dimensional rotating stage chase the movement of the retroreflector, and a change in the distance from each interferometer to the retroreflector is continuously measured.

By using LTS, the target retroreflector is directly measured to conform Abbé's principle. Considering from this fact, LTS is capable of performing more accurate measurement than CMM. By using LTS, not only we can measure three-dimensional coordinate, but we can calibrate ordinary CMMs. We also applied LTS to the performance evaluation of industrial robots.\textsuperscript{2,3}

Applying the principle of trilateration, the coordinate of the retroreflector is calculated from the measurement values of the laser interferometers. The arrangement of all stations and the initial length between each station to the retroreflector should be known values (we call these values "system parameters"). For this purpose, we used the fourth station although three stations are required in minimum to determine the three-dimensional coordinate.\textsuperscript{4} Taking advantage of the redundancy in the measurement created by the fourth station, all system parameters are determined inversely from the measurement results of LTS. This algorithm is called "self-calibration".

Further author information -
T.T.(correspondence): Email: takat@nrlm.go.jp; Telephone: +81-298-54-4041; Fax: +81-298-54-4042
M.G.: Email: goto@nrlm.go.jp
T.K.: Email: kurosawa@nrlm.go.jp
Y.T.: Email: tanimura@nrlm.go.jp; Telephone: +81-298-54-4151; Fax: +81-298-54-4008
Y.K.: Email: koseki@mel.go.jp; Telephone: +81-298-58-7273; Fax: +81-298-58-7201
To do the self-calibration algorithm, there exists a restriction on the arrangement of stations in laser trilateration. In this paper, this restriction will be discussed theoretically.

Finally a preliminary experiment was made in which this restriction is conformed. The measurement error in this stage is about 40 μm for a 1 m measurement.

2. SELF-CALIBRATION ALGORITHM

In our LTS, the principle of trilateration was adopted. When three reference points whose arrangement has been already known exist, the coordinate of a target can be determined by the length between each reference point (i.e. the position of the laser tracker) and the target.

In actual measurements, it is almost impossible to measure the arrangement of stations with an accuracy of sub-micrometer. Moreover laser interferometers can detect only changes in distance of the target, therefore initial length between each station and the target retroreflector should be measured by other means at the beginning of each measurement. The arrangement of stations and the initial length between each station and the target are called system parameters.

To overcome this difficulty, we applied a self-calibration algorithm. To determine three-dimensional coordinate based on laser trilateration, three stations are required in minimum. On the other hand, we made use of the fourth stations. This excess one station gives redundancy to the measurement, and therefore it enables to self-determine the system parameters only from the measurement values without any other devices.

We denote the positions of stations as \( S_1 \sim S_4 \). Without loosing generality, \( S_1 \sim S_2 \) can be assumed as \( S_1 \) \((0, 0, 0)\), \( S_2 \) \((X_2, 0, 0)\), \( S_3 \) \((X_3, Y_3, 0)\), and \( S_4 \) \((X_4, Y_4, Z_4)\), namely \( S_1 \) is located at the origin, \( S_2 \) is on the \( X \) axis, and \( S_3 \) lies on the \( XY \) plane.
plane. Then the initial length of the $i$th station is expressed as $l_i$ ($i = 1 - 4$), and at the $j$th measurement point the path change measured by the $i$th station is denoted as $m_{ij}$ ($j = 1 - n$, $n$ is the number of measurement points).

There are three unknown variables (the $X$, $Y$, and $Z$ coordinates) at each measurement point, while four measurement values are acquired by four stations. In other words, one redundant data is collected at each measurement point. The position of the target retroreflector can be calculated by any three of the four measurement values. The length between the $i$th station and the position calculated by three measurement values other than $i$th station is written as $k_i$. We define the residual $R_j$ for the $j$th measurement point as

$$R_j = \left( \sum_{i=1}^{4} \left[ k_{ij} - (l_i + m_{ij}) \right]^2 \right)^{1/2},$$

and then also define the sum of square root of $R_j$ as $\text{Res}$ which should be minimized to determine the system parameters.

$$\text{Res} = \sum_{j=1}^{n} R_j^2 \rightarrow \min.$$

There are six variables concerning the arrangement of the laser trackers, and three variables concerning the initial length of the interferometers (since one of the initial length can be calculated from other three initial lengths). Consequently when the number of measurement points is equal to or more than nine, all system parameters can be self-calibrated only from measurement data. Actually this calculation is done by using Gauss-Newton method which is one of a successive approximation method.

### 3. RESTRICTION ON THE ARRANGEMENT OF STATIONS

The self-calibration algorithm enables us to determine the system parameters only from the measurement data. No precise setup for measurements is required. As far as we explained in section 2, all stations can be located at arbitrary positions. Nevertheless there is a restriction on the arrangement of the laser trackers. If the restriction is not satisfied, self-calibration algorithm is impossible, and consequently we cannot determine the position of the laser trackers.

In this section, we investigate this restriction theoretically. The relationships among the system parameters, the coordinate of the target, and measurement values are written as follows;

$$X^2 + Y^2 + Z^2 = (l_1 + m_1)^2,$$

$$Y = (l_1 + m_1)^2 - X^2 - Z^2.$$  

These four equations are satisfied simultaneously. The substitution of equation (3-4) into the other three equations yields

$$\begin{pmatrix} x_2 & 0 & 0 \\ x_3 & y_3 & 0 \\ x_4 & y_4 & z_4 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} (l_2 + m_2)^2 - (l_1 + m_1)^2 - x_2^2 \\ (l_3 + m_3)^2 - (l_1 + m_1)^2 - x_3^2 - y_3^2 \\ (l_4 + m_4)^2 - (l_1 + m_1)^2 - x_4^2 - y_4^2 - z_4^2 \end{pmatrix}.$$  

(4)
To solve this equation for $X$, $Y$ and $Z$, the inverse matrix of the $3 \times 3$ matrix on the left-hand side should exist, namely,

$$
M^{-1} = \begin{pmatrix}
\frac{1}{x_2} & 0 & 0 \\
-x_3 & \frac{1}{y_3} & 0 \\
-x_4 + \frac{x_3y_4}{y_3z_4} & -\frac{y_4}{y_3z_4} & \frac{1}{z_4}
\end{pmatrix} .
$$  \hfill (5)

If the denominators of any elements equal to zero, the inverse matrix does not exist. In this case, equations (3-1) – (3-4) are not independent of each other, and consequently the self-calibration algorithm of the system parameters cannot be executed.

There are three cases where this situation occurs. The first case is $x_2 = 0$, i.e. two laser trackers $S_1$ and $S_2$ overlap each other. The second case is $y_3 = 0$, i.e. three laser trackers $S_1$, $S_2$, and $S_3$ exist on $X$ axis. And the third case is $z_4 = 0$, i.e. the fourth station $S_4$ lies in the $XY$ plane. Since the orientation of the coordinate axes in three-dimensional space is not absolute, these three cases can be unified into the third case. Therefore, the restriction on the arrangement of laser trackers is that

"no laser tracker must exist on the plane defined by the other three laser trackers." \hfill (*)

Now we will investigate this situation in more detail. When $z_4$ equals to zero, $z_4$ cannot be substituted into equation (5). Substituting $z_4$ into equation (3-4) and eliminating $X$, $Y$, and $Z$ from equations (3-1) – (3-4)

$$
A(t_1 + m_1)^2 + B(t_2 + m_2)^2 + C(t_3 + m_3)^2 - (A + B + C)(t_4 + m_4)^2 = D ,
$$  \hfill (6)

is derived, where

$$
A = -y_3x_4 + x_3y_4 - x_2y_4 + x_2y_3 , \hfill (6-1)
$$

$$
B = y_3x_4 - x_3y_4 , \hfill (6-2)
$$

$$
C = x_2y_4 , \hfill (6-3)
$$

$$
D = x_4(x_2x_3x_4 - y_3x_4^2 + x_3^2y_4 + y_3^2y_4 - x_2x_3y_4 - y_3y_4^2) . \hfill (6-4)
$$

Equation (6) is always satisfied, regardless of whether or not the system parameters conform to restriction (*)

Coefficients $A \sim D$ include only parameters concerning the position of the laser trackers. There are five position parameters, namely $x_2$, $x_3$, $y_3$, $x_4$, and $y_4$, while the number of the coefficients is four, namely $A$, $B$, $C$, and $D$. Consequently an infinite number of incorrect combination of the coefficients, i.e. $A' \sim D'$, can exist. For these combinations, $A/D$, $B/D$, and $C/D$ are same as $A'/D'$, $B'/D'$, and $C'/D'$ respectively. For example, an arrangement (I)

$$
S_1 ( \begin{array}{ccc} 0.0000, & 0.0000, & 0.0000 \end{array} ) ,
$$

$$
S_2 ( \begin{array}{ccc} 1000.0000, & 0.0000, & 0.0000 \end{array} ) ,
$$

$$
S_3 ( \begin{array}{ccc} 0.0000, & 400.0000, & 0.0000 \end{array} ) ,
$$

$$
S_4 ( \begin{array}{ccc} 500.0000, & 400.0000, & 0.0000 \end{array} ) ,
$$

$$
R ( \begin{array}{ccc} 0.0000, & 800.0000, & 2000.0000 \end{array} ) ,
$$
and an arrangement (II)

\[
\begin{align*}
S_1 & \quad (0.0000, 0.0000, 0.0000), \\
S_2 & \quad (997.9511, 0.0000, 0.0000), \\
S_3 & \quad (-1.0255, 397.2130, 0.0000), \\
S_4 & \quad (497.9501, 397.2130, 0.0000), \text{ and}
\end{align*}
\]

R \quad (-2.0596, 802.8124, 1998.8717),

have the same initial length from each laser tracker to the target retroreflector as

\[
\begin{align*}
l_1 &= 2154.0659, \\
l_2 &= 2374.8684, \\
l_3 &= 2039.6078, \text{ and}
\end{align*}
\]

\[
l_4 = 2100.0000.
\]

For both arrangements, the ratios of A/D, B/D, and C/D in equation (6) are same each other. Additionally for instance, a point (200.0000, 800.0000, 2000.0000) in arrangement (I) and a point (198.3597, 803.3298, 1998.8284) in arrangement (II) show the same measurement results, i.e. \( m_1 = 9.2648, \ m_2 = -77.0434, \ m_3 = 9.7823, \) and \( m_4 = 38.4472. \)

4. EXPERIMENTAL INSTRUMENT

A photograph of the laser tracker is shown in figure 2. A small laser interferometer head is connected to a stabilized He-Ne laser head with an optical fiber. The interferometer head is mounted on a two-dimensional (horizontal and vertical) stage. The laser beam emitted from the interferometer head illuminates the target, and then is reflected on the target to the reverse direction. The returning beam is divided into two beams in the interferometer head. One is used for measuring the movement of the target, and the other beam is guided to a quadrant photo-diode. The electric signal from the diode is inputted to a servo controller of the stage so as the laser beam chases the movement of the target continuously.

The laser beam must be reflected on the surface of the target to the reverse direction even if the beam incidents from any directions. A mirror which has this feature is called a retroreflector, and we used a cat's eye retroreflector.

5. EXPERIMENT

The cat's eye retroreflector is fitted on a coordinate measuring machine (CMM), which we borrowed from Tokyo Seimitsu company, instead of a normal probe head, and the position of it was measured.

Four laser trackers were placed on the top plate of a CMM. The arrangement of the stations were determined so as to conform the restriction (*) as shown in figure 3. Namely three laser trackers resided in a plane parallel to the XZ plane, and the fourth laser tracker was placed out of the plane.
The area of 800 mm (X) x 240 mm (Y) x 480 mm (Z) was measured, and the orientation and the origin of the coordinate is indicated also in figure 3.

The cat's eye retroreflector was moved every 200 (X), 60 (Y), and 120 mm (Z) in the measurement volume, consequently the coordinate of 125 points were measured in the first run. Exactly same measurements were performed in the second run. Finally the measurement volume was shifted by +50 mm in the Y direction, and in this measurement a part of the measurement volume was blocked by the holder of the cat's eye, so the measurement points was 120.

6. EXPERIMENTAL RESULT

To evaluate the coordinate of the measurement result, the coordinate system of the LTS should be rotated to that of the CMM. To avoid this conversion, we compared the distances between two measurement points to nominal values.

Three measurements showed quite similar results, hence we will discuss only the result of the first measurement. Although the measurement result by the CMM has measurement errors, we assumed measurement results by the CMM did not contain any errors.

Figure 4 shows the relationship between the measurement length and the measurement error for each axis, in which the linear least-square fitting lines are drawn. It is clearly observed that there is a systematic error which increases with the measurement length. This fact can be understood as that the length scale of each axis of the CMM does not coincide with that of the LTS.

When the wavelength of the laser changes by some reasons, this type of error occurs. However the wavelength change leads the measurement error of same amount for all axis, i.e. tangent of three lines should be same. In this experiment, we used a stabilized He-Ne laser with a frequency instability of less than 10⁻⁷, therefore the influence of the laser itself can be negligible.

Although the measurement error for the measurement length of zero should naturally be zero, in this experiment the three regression lines drawn in figure 4 do not cross the origin. This fact indicates that the measurement length and the measurement error have a nonlinear relationship.

There are many causes which contribute to the measurement error. We think the largest one is an estimation error of the system parameters (namely the position of the stations and the initial position of the target retroreflector). The result of the self-calibration algorithm does not have any measurement error theoretically and also in computer numerical simulations. However in actual calculation, the successive calculation can converge at a value which slightly differs from the correct value.

The measurement error of this instrument is considerably large: 40 μm for 1 m measurement in maximum. In this stage, this instrument can be applied only for some particular purpose, but cannot be used for the purpose of calibrating CMMs.
7. CONCLUSIONS

We have developed a laser tracking interferometer system which is capable of measuring a three-dimensional coordinate. The redundancy produced by the use of the fourth laser tracker enables to self-determine all system parameters (the position of the laser trackers and the initial position of the retroreflector).

Because the LTS satisfies the Abbé's principle, it can perform more accurate measurement than CMM in principle. Additionally, using LTS the coordinate is calculated from the laser wavelength, therefore its measurement is directly related to a higher level of length standard. Calibration of CMM using LTS is more ideal than that using artifacts like step or ball gauges.

In this system the system parameters is self-calibrated, and consequently we can place the laser trackers at almost arbitrary positions. However there is one restriction on the arrangement of the laser trackers, namely no laser tracker must exist on the plane defined by the other three laser trackers. To find the best arrangement, further investigation should be needed.

With satisfying the restriction, preliminary experiment have been performed. Four laser trackers were placed on the top plate of a CMM, and the coordinate of 125 points were measured by both CMM and LTS. Comparison of measurement values by two instruments result in 40 μm deviation for 1 m measurement each other.

In order to improve the measurement accuracy of the LTS, the optical and mechanical system of the laser tracker should be refined and the self-calibration algorithm should be investigated in more detail.

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9. REFERENCES